Chaos Driven Decay of Nuclear Giant Resonances: Route to Quantum Self-Organization

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Abstract

The influence of background states with increasing level of complexity on the strength distribution of the isoscalar and isovector giant quadrupole resonance in ^{40}Ca is studied. It is found that the background characteristics, typical for chaotic systems, strongly affects the fluctuation properties of the strength distribution. In particular, the small components of the wave function obey a scaling law analogous to self-organized systems at the critical state. This appears to be consistent with the Porter-Thomas distribution of the transition strength.

PACS numbers: 05.45.+b, 24.30.Cz, 24.60.-k, 24.60.Lz, 47.52.+j

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Nuclear physics has contributed significantly to the recent progress in understanding the chaotic aspects of nonlinear dynamical systems, especially in the context of classical versus quantum correspondence. The nucleus is particularly well suited for such studies due to the intrinsic quantum nature of the nuclear many-body problem on the one hand, and the wealth of experimental data on the other. The chaotic nature of the nucleus is by now well documented empirically [1] and seems natural, bearing in mind the many-body character of the nucleus and the complicated form of the nucleon-nucleon interaction. Still, however, nuclei reveal many regular, collective phenomena and this coexistence of chaos and collectivity is a challenging problem for quantitative study [2]. Especially interesting in this respect are the nuclear giant resonances which carry a large fraction of the total transition strength and are located many MeV above the ground state, in the energy region which is expected to be dominated by chaotic dynamics.

The giant resonance, as a short time phenomenon, involves simple configurations of one-particle one-hole (1p-1h) type. Chaos may influence the subsequent decay of these components which occurs on longer time scales and gradually evolves to more and more complex configurations. Eventually, the initial energy deposited in the nucleus is redistributed over all available degrees of freedom and the limit of the compound nucleus is reached. This is the limit of fully developed chaos and, as a result, quantum stochastic methods based on the random matrix theory [3] or, alternatively, molecular dynamical approaches generating chaotic behavior [4], prove appropriate. The process of giant resonance formation and its subsequent decay towards the compound nucleus occurs in a closed system and the most basic approach is in terms of a single Hamiltonian acting in a rich enough Hilbert space such that the relevant degrees of freedom are included. This also provides the most natural scheme for the coupling between a collective state and the complex background. In quantum mechanical terms one can then speak about

the large and the small components of the nuclear wave function [5]. It is the purpose of the present paper to work out such a scheme, to identify which ingredients are relevant, to study the role of chaos on the giant resonance physical observables and finally, from a more general perspective, to contribute to the understanding of the universal aspects of the coexistence between collectivity and chaos in small many-body quantum systems.

We start from the nuclear Hamiltonian which, in second quantized form, reads as

$$\hat{H} = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{2} \sum_{ij,kl} v_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k. \tag{1}$$

The first term denotes the mean field which represents its most regular part [6] while the second term is the residual interaction. Clearly a diagonalization of $\hat{H}$ in the full Hilbert space of n-particle n-hole excitations is numerically prohibitive, and may not be necessary. Our recent study [7] of level statistics in the subspace generated by two-particle two-hole (2p-2h) excitations has shown that it is sufficient to diagonalize the Hamiltonian within a truncated subspace of 1p-1h and 2p-2h states [8]:

$$|1\rangle \equiv a_p^\dagger a_h |0\rangle; \quad |2\rangle \equiv a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1} |0\rangle \tag{2}$$

For the present discussion it is more transparent to prediagonalize v in the 1p-1h and 2p-2h subspaces such that

$$|\tilde{1}\rangle = \sum_1 C_1^{\tilde{1}} |1\rangle; \quad |\tilde{2}\rangle = \sum_2 C_2^{\tilde{2}} |2\rangle \tag{3}$$

Then the Schrödinger equation takes the following form

$$\begin{pmatrix} E_{\tilde{1}} & A_{\tilde{1}\tilde{2}} \\ A_{\tilde{2}\tilde{1}}^* & E_{\tilde{2}} \end{pmatrix} \begin{pmatrix} X_{\tilde{1}}^N \\ X_{\tilde{2}}^N \end{pmatrix} = E_N \begin{pmatrix} X_{\tilde{1}}^N \\ X_{\tilde{2}}^N \end{pmatrix} \tag{4}$$

where $E_{\tilde{1}}$ and $E_{\tilde{2}}$ denote the energy eigenvalues in the 1p-1h and 2p-2h subspaces respectively and $A_{\tilde{1}\tilde{2}} = \sum_{12} C_1^{\tilde{1}} \langle 1|v|2\rangle C_2^{\tilde{2}}$ mediates the coupling between these two spaces.

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The solutions of Eq. (4) yield the transition strength distribution $S_F(E)$ in response to an external field $\hat{F} = \sum_{ij} F_{ij} a_i^\dagger a_j$ as

$$S_F(E) = \sum_{\tilde{1},N} |X_{\tilde{1}}^N \langle 0|\hat{F}|\tilde{1}\rangle|^2 \delta(E - E_N) \tag{5}$$

with its energy moments being defined as

$$\langle E^n \rangle = \int dE S_F(E) E^n / \int dE S_F(E). \tag{6}$$

For the specific case of the quadrupole response in ^{40}Ca, considered here, we have chosen the mean field and residual interaction as in ref. [9] including two major shells above and below the Fermi level. Our study is based on an explicit diagonalization in Eq. (4) which involves more than 11,000 states out of which only 26 are 1p-1h states. Computational restrictions require to limit the number of 2p-2h states. It turns out that, including those up to 50 MeV excitation energy, suffices for a realistic description of the measured response function [10]. This yields altogether 3014 2^+ states which is numerically manageable. The results displayed in Fig. 1 and Fig. 2 yield a mean excitation energy $\langle E \rangle$ of 30.84 MeV for the isovector and 24.45 MeV for the isoscalar transitions, independent of the mixing with 2p-2h states. The latter can be easily understood by writing the mean energy as $\langle E \rangle = \langle 0|\hat{F}^\dagger[\hat{H}, \hat{F}]|0\rangle$ and realizing that $|0\rangle$ is given by a Slaterdeterminant of occupied single-particle states. For the energy dispersion, $\sigma = (\langle E^2 \rangle - \langle E \rangle^2)^{1/2}$, we obtain 3.41 MeV and 4.28 MeV when only 1p-1h excitations are considered. Allowing for the coupling to 2p2h-states these values are increased to 5.40 MeV and 5.42 MeV, respectively, independent of whether effects of the residual interaction in the 2p-2h subspace is included or not. The latter can be deduced from the form of $|0\rangle$ and the fact that $\langle E^2 \rangle = -\langle 0|[\hat{H}, \hat{F}^\dagger][\hat{H}, \hat{F}]|0\rangle$. Motivated by our previous results [7] on the level fluctuations in the prediagonalized 2p-2h space

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we distinguish three cases: (1) no residual interaction in this space (the corresponding strength distribution is shown in Figs. 1b and 2b) for which the nearest-neighbor spacing distribution is sharply peaked near zero, because of degeneracies, (2) the inclusion of particle-particle and hole-hole two-body matrixelements (Figs. 1c and 2c) which results in a Poissonian distribution, characteristic of the known universality class of generic integrable systems [11] (3) the use of the full residual interaction (Figs. 1d and 2d) which yields the fluctuations of the Gaussian orthogonal ensemble (GOE) [1] characteristic of classically chaotic systems. All three cases introduce significant modifications of the 1p-1h 'doorway' strength distribution (notice the change in magnitude of the transition matrixelements) resulting in a gradual reduction of the large components accompanied by a simultaneous amplification of the smaller ones when going from case (1) to case (3). The isovector response is affected more than the isoscalar one especially when going from (2) to (3) (Figs. 1c and 1d and Figs. 2c and 2d).

Of course, what is important for the degree of mixing between the spaces spanned by $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ is not only the spectral properties of the eigenenergies in $|\tilde{2}\rangle$ but also the distribution of the coupling matrix elements $A_{\tilde{1}\tilde{2}}$. This distribution can be influenced by the degree of coherence in the wave packet $|F\rangle = \hat{F}|0\rangle$, initially formed by an external field. The motion in the isoscalar case is more coherent than in the isovector case since protons and neutrons move in phase. Therefore, the isoscalar state is expected to be more resistive against decay than its isovector counterpart [8]. This tendency is also seen from Fig. 3 which displays the number of matrix elements $A_{F\tilde{2}}$ of given magnitude, connecting the state $|F\rangle$ and the states $|\tilde{2}\rangle$. In the cases referred to as (1) and (2) above, additional selection rules lead to large degeneracies and consequently the corresponding distributions sharply peak at zero. When the full residual interaction is used in generating the vectors $|\tilde{2}\rangle$ (case (3)) these selection rules are removed and there

are no vanishing coupling matrix elements any more. In the isovector case the wings of distribution seem to be consistent with a Gaussian as one would expect for a random process. In the isoscalar case, however, $A_{F\tilde{2}}$ remains more localized around small values and a Gaussian fit to the wings is unsatisfactory.

The analysis of the distribution of the mixing matrix elements is consistent with the results seen in Figs. 1 and 2; the isovector strength distribution is more affected by the complex background. Another and perhaps the most interesting effect is that the isovector strength is distributed much more uniformly, not only in excitation energy but also in magnitude. A bit of imagination may even suggest a certain kind of self-similarity regarding the clustering and the relative size of the transitions. The picture in Fig. 1d becomes reminiscent of a self-organized system at its critical state [12, 13] where the equilibrium balance reduces the dimensionality. This observation finds confirmation in more quantitative terms. Fig. 4 shows the total number N of transitions of magnitude smaller than a given threshold value S_{th}, as a function of S_{th}. For the isovector resonance in the chaotic case (Fig. 1d) we find, except for the largest transitions, a scaling law of the form $N \sim S_{th}^{\alpha}$ ($\alpha \approx 0.50$) (indicated by the straight line fit in the upper part of Fig. 4) which indeed signals a reduction of dimensionality. The two nonchaotic cases (Figs. 1b and c) display a more complicated behavior. It is interesting to notice that a similar scaling applies also to the isoscalar resonance (lower part of Fig. 4) even though this is not so obvious from Fig. 2d. This time, however, the scaling interval is somewhat shorter and $\alpha \approx 0.48$.

The strength distribution can be considered as an attractor for the decay process starting out of equilibrium. It is nothing but the Fourier transform of the time correlation function $\langle F(0)|F(t)\rangle$ which, in the form of an envelope [14], describes the process of gradual convergence to such an attractor and, thus, resolves it. This sets the parallel

to a procedure [15] which resolves the self-similarity and the scale invariance in classical chaos. This analogy provides further arguments for interpreting the above scaling law as another manifestation of '1/f'-type behavior [16]. So far, such a behavior has been identified mostly on the classical level [17] for a variety of observables and models. In the present case of an 'avalanche' of the decaying giant resonance it is reflecting the chaotic properties of a strictly quantum mechanical phenomenon.

The influence of chaos on the strength distribution is believed [18, 19] to manifest itself in a Porter-Thomas distribution [20] of transition strengths maximizing the entropy of the strength distribution [18]. Converting the appropriate (differential) Porter-Thomas distribution to a cumulative allows a direct comparison with our calculated distributions. The results are given by the thin solid lines in Fig. 3. They show the same scaling behavior with $\alpha = 1/2$ exactly as can be easily seen from the analytical form of the Porter-Thomas distribution. One thus finds an impressive consistence. The strength distributions in Figs. 1 and 2 represent the coexistence of collectivity and chaos and, therefore, the amplified abundance of large components as compared to a purely random process is natural. Being more coherent, for the isoscalar resonance the onset of the scaling regime is delayed. It is also interesting to see that even a fully random process, as represented by the Porter-Thomas distribution, develops a trace of such a delay. The possibility of a similar two-phase behavior has been identified [21] in random cascade models.

express his thanks to the Alexander von Humboldt Foundation for a fellowship.

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Figure Captions

Fig. 1 The isovector quadrupole strength distribution in ^{40}Ca: (a) no coupling to the 2p-2h subspace (b) no residual interaction in 2p-2h subspace (c) including only particle-particle and hole-hole matrixelements in the diagonalization of the 2p-2h subspace (d) diagonalization of the full residual interaction in the 2p-2h subspace.

Fig. 2 Same as Fig. 1 but for the isoscalar quadrupole strength distribution.

Fig. 3 The distribution of coupling matrixelements of the isovector and isoscalar wave packets $|F\rangle$ to 2p-2h excitations: (a) with no residual interaction in the 2p-2h subspace, (b) including only pp- and hh matrixelements in this space, (c) diagonalizing the residual interaction fully. The full lines denote a Gaussian fit to these distributions.

Fig. 4 The total number N of transitions of given strength below a threshold value S_{th} as a function of S_{th}. The open triangles refer to the case (b), the open squares to the case (c) and thick dots to the case (d) of Figs. 1 and 2, respectively. The solid line indicates the best fit to the later case. The thin solid lines represents the same quantity determined from a Porter-Thomas distribution.

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